Problem 1. For the rectangular column section given below, determine:

a) the balanced compressive load, \( \phi P_b \), the balanced moment, \( \phi M_b \), and the balanced eccentricity, \( e_b \). (Answer: \( \phi P_b = 284 \text{ Kips}, e_b = 15.3 \text{ in.} \))
b) The load capacity, \( \phi P_n \), when the eccentricity is \( e = 6 \text{ in.} \). (Answer: \( \phi P_b = 565 \text{ Kips} \))
c) The load capacity, \( \phi P_n \), when the eccentricity is \( e = 24 \text{ in.} \). (Answer: \( \phi P_b = 159 \text{ Kips} \))

Column section: \( f'_c = 5 \text{ Ksi}, f_y = 60 \text{ Ksi}, b = 16 \text{ in}, h = 16 \text{ in}, A_s = A_s' = 5 \text{ No. 10}. \)

Problem 2: Determine the load capacity, \( \phi P_n \), for the column section shown, when the eccentricity is \( e_y = 8 \text{ in.} \). Use \( f'_c = 3 \text{ Ksi} \) and \( f_y = 60 \text{ Ksi} \). (Answer: \( \phi P_b = 709 \text{ Kips} \))

Problem 3: Design a rectangular column section to support the ultimate load, \( P_u = 310 \text{ Kips} \), and moment, \( M_u = 310 \text{ Kips-ft} \). Determine \( A_s, A_s' \), and \( h \), then choose the adequate bars considering \( A_s = A_s' \). The final total steel ratio, \( \rho_s \), should be close to 2\%. (One possible solution: \( h = 20 \text{ in}, A_s = A_s' = 3 \text{ No. 9} \))

Problem 4 (Not for Fall 06!!): Using the same column section of Problem 2, with \( e_y = 8 \text{ in} \) and \( e_x = 6 \text{ in} \), determine the load capacity \( \phi P_n \). Use \( f'_c = 3 \text{ Ksi} \) and \( f_y = 60 \text{ Ksi} \). Using the equivalent eccentricity method.

Problem 5 (Not for Fall 06!!): Design a column for an ultimate load \( P_u = 800 \text{ Kips} \), with eccentricities \( e_y = 8 \text{ in} \) and \( e_x = 6 \text{ in} \).

Problem 6: For the rectangular column section, determine the load capacity \( \phi P_n \) when the eccentricity is \( e = 6 \text{ in} \).
Given: \( f'_c = 5 \text{ ksi}; b = 14 \text{ in}; h = 20 \text{ in}; A_s = A_s' = 4 \text{ No. 9}; f_y = 60 \text{ ksi}; d' = 2.5 \text{ in} \).
Solution:
We will use statics and equilibrium to solve the problem. The strain and internal forces diagrams are shown below:

The equations are:

\[
\frac{0.003}{c} = \frac{\varepsilon_1}{c - 2.5} = \frac{\varepsilon_2}{17.5 - c}
\]

\[
P_n = F_{s1} + C_c - F_{s2} = f_{s1}(4) + 0.85(5)(14a - 4) - f_{s2}(4)
\]

\[
M_n = F_{s1}(10 - 2.5) + C_c (10 - a / 2) + F_{s2}(17.5 - 10) =
\]

\[
f_{s1}(4)(10 - 2.5) + 0.85(5)(14a - 4)(10 - a / 2) + f_{s2}(4)(17.5 - 10)
\]

\[
c = 0.80 \times a
\]

\[
e = \frac{M_n}{P_n} = 6 \text{ in}
\]

Let’s assume: \(\varepsilon_1 > \varepsilon_y \rightarrow f_{s1} = 60 \text{ ksi}\)

\(\varepsilon_2 < \varepsilon_y \rightarrow f_{s2} = E\varepsilon_2 = 29000\varepsilon_2\)

Solving the equations we get:
\(c = 14.66 \text{ in}\)
\(P_n = 853.58 \text{ K}\)

Verification. For \(c = 14.66 \text{ in.}\), we get:

\(\varepsilon_1 = 0.00249 > \varepsilon_y \) (OK)

\(\varepsilon_2 = 0.00058 < \varepsilon_y \) (OK)

Since \(\varepsilon_i = 0.00058 < \varepsilon_y\), the section is compression controlled. Therefore \(\phi = 0.65\) and

\(\phi P_n = 0.65(853.64 \text{ Kips}) = 555.52 \text{ Kips}\)
Problem 7: Determine the load capacity $\phi P_n$ for the column section shown, when the eccentricity is $e = 8$ in. Use $f'_c = 4$ ksi and $f_y = 60$ ksi.

Solution: Similar to the previous problem, we will solve this by using statics and equilibrium. The difference now is, we have three layers of reinforcement. However, the procedure, although longer, is very similar. The equations are:

\[ P_n = F_{s1} + C_c + F_{s2} - F_{s3} = f_{s1}(3) + 0.85(4)(14a - 5) + f_{s2}(2) - f_{s3}(3) \]

\[ M_n = F_{s1}(10 - 2.5) + C_c(10 - a/2) + F_{s3}(17.5 - 10) = f_{s1}(3)(10 - 2.5) + 0.85(4)(14a - 5)(10 - a/2) + f_{s3}(3)(17.5 - 10) \]

\[ c = 0.85a \]

\[ e = \frac{M_n}{P_n} = 8 \text{ in} \]

Let's assume: $\varepsilon_{s1} > \varepsilon_y \Rightarrow f_{s1} = 60$ ksi

$\varepsilon_{s2} < \varepsilon_y \Rightarrow f_{s2} = E\varepsilon_{s2} = 29000\varepsilon_{s2}$

$\varepsilon_{s3} > \varepsilon_y \Rightarrow f_{s3} = 60$ ksi

Solving the equations we get:
\[ c = 14.28 \text{ in} \]
\[ P_n = 613.10 \text{ K} \]

Verification. For \( c = 14.28 \text{ in.} \), we get:
\[ \varepsilon_{s1} = 0.00247 > \varepsilon_y \text{ (OK)} \]
\[ \varepsilon_{s2} = 0.0009 < \varepsilon_y \text{ (OK)} \]
\[ \varepsilon_{s3} = 0.00067 < \varepsilon_y \text{ (Not OK!)} \]

We’ll have to make other assumption.

Let’s assume: \( \varepsilon_{s1} > \varepsilon_y \rightarrow f_{s1} = 60 \text{ ksi} \)
\[ \varepsilon_{s2} < \varepsilon_y \rightarrow f_{s2} = E\varepsilon_{s2} = 29000\varepsilon_{s2} \]
\[ \varepsilon_{s3} < \varepsilon_y \rightarrow f_{s3} = E\varepsilon_{s3} = 29000\varepsilon_{s3} \]

Solving the equations we get:
\[ c = 12.09 \text{ in} \]
\[ P_n = 565.11 \text{ K} \]

Verification. For \( c = 12.09 \text{ in.} \), we get:
\[ \varepsilon_{s1} = 0.00240 > \varepsilon_y \text{ (OK)} \]
\[ \varepsilon_{s2} = 0.00052 < \varepsilon_y \text{ (OK)} \]
\[ \varepsilon_{s3} = 0.00134 < \varepsilon_y \text{ (OK)} \]

Since \( \varepsilon_t = 0.00134 < \varepsilon_y \), the section is compression-controlled. Therefore \( \phi = 0.65 \) and
\[
\phi P_n = 0.65(565.11 \text{ Kips}) = 367.32 \text{ Kips}
\]
SOLUTION TO PROBLEM 4 (NOT FOR FALL 06!!):

Using the same column section of Problem 2, with $e_y = 8 \text{ in}$, and $e_x = 6 \text{ in}$, determine the load capacity $\phi P_u$. Use $f' = 3 \text{ Ksi}$ and $f_y = 60 \text{ Ksi}$. Using the equivalent eccentricity method.

![Column Section Diagram](image)

**Figure 1**

Solution:

\[
\begin{align*}
\frac{e_x}{l_x} &= \frac{6 \text{ in}}{16 \text{ in}} = 0.375 \\
\frac{e_y}{l_y} &= \frac{8 \text{ in}}{24 \text{ in}} = 0.333
\end{align*}
\]

Therefore: $\frac{e_x}{l_x} > \frac{e_y}{l_y}$; and the eccentricity is along $x$ axis.

\[
e_{ox} = e_x + \frac{\alpha e_y l_x}{l_y}
\]  \hspace{1cm} \text{Eqn. 1}

Since we can’t compute $\alpha$ at this time for it depends on the $P_u$ value, we assume a value of $\alpha = 0.7$

From $e_{ox} = e_x + \frac{\alpha e_y l_x}{l_y}$ \hspace{1cm} \text{Eqn. 1}:

\[
e_{ox} = 6 \text{ in} + \frac{(0.7)(8 \text{ in})(16 \text{ in})}{24 \text{ in}} = 9.733 \text{ in}
\]

Now the problem is reduced to the same column, the same load but with an eccentricity only along $x$ axis. See Figure 2. It doesn’t matter that the $P_u$ is located “outside” of the column. It is not its actual location and besides that position is due to a “high” value of the moment acting on the column.
a. We can use the approximate interaction diagrams that are available on Design Manuals. For instance, let’s use the diagrams given by the textbook. Remember: the tables given by the textbook are only for \( f' = 3,000 \text{ psi} \) and \( f_y = 60,000 \text{ psi} \). Otherwise, you cannot use those tables. Remember also the two reinforcement arrangements that the textbook covers. In this case we have to use the “reinforcement equally distributed on all four sides.”

First, we compute the value of \( g \). Remember: \( g \) must be measured along the direction where the eccentricity lies. Remember also that \( h \) is measured along the same direction. In our case:

\[
\gamma = \frac{16 - 2(1.5) - 2(3/8) - 10/12}{16} = 0.714
\]

Other values needed are: \( \frac{e}{h} = \frac{9.733}{16} = 0.608 \), and \( \rho_g = \frac{12(1.27)}{(24)(16)} = 0.0397 \)

For \( \gamma = 0.60 \), \( e/h = 0.608 \) and \( \rho_g = 0.0397 \), we use Fig A-9 (from Textbook) to approximately obtain: \( \frac{\phi P_n}{bh} = 0.8 \text{ ksi} \)

For \( \gamma = 0.75 \), \( e/h = 0.608 \) and \( \rho_g = 0.0397 \), we use Fig A-10 (from Textbook) to approximately obtain: \( \frac{\phi P_n}{bh} = 0.95 \text{ ksi} \)

By linear interpolation we determine that for \( \gamma = 0.714 \), \( \frac{\phi P_n}{bh} = 0.914 \text{ ksi} \)

From which: \( \phi P_n = 0.914(16)(24) = 350.976 \text{ kips} \)

\[\boxed{\phi P_n = 350.98 \text{ kips}}\]

Now we have to check whether our guess value of \( \alpha \) was good.

To compute \( \alpha \):

\[
\frac{P_u}{f' A_g} = \frac{350.98}{3(24)(16)} = 0.305 < 0.4
\]
Therefore: \[ \alpha = \left( 0.5 + \frac{P_u}{f'_c A_g} \right) \frac{f_y + 40,000}{100,000} = (0.5 + 0.305) = 0.805 > 0.6 \]

We could refine our analysis using this new value of \( \alpha \). This will not be done here.

Another option is the compute the interaction diagram for the given column. This has been covered in other classes, and it will not be shown here.